Advanced Speed Control of a Three Phase Induction Machine Based on Rotor and Stator Flux Orientation Utilizing ANNs Technique*

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Abstract

This paper presents a design approach to enhance speed estimation and control for three phase induction machine drive system by the implementation speed sensor less field orientation technology. A Direct Stator Flux Oriented method integrated by Artificial Neural Network is followed to evaluate the machine speed and Indirect Rotor Flux Oriented method with Hysteresis Band Pulse Width Modulation is employed to for the system control. This control model allows sinusoidal currents to drive the motor with producing less speed and torque ripples, which minimizes the acoustic noise provides by field-weakening operation at high and low-speeds.

Computer simulation that is using Mat lab\Simulink software shows acceptable controlled performance results for forward and reverses for motoring and generating operation. At the machine is motoring, speed range of (0- ±1250 rpm) with mechanical load torque variation up to (7 N-m) during steady state and dynamic condition.

Keywords: Induction machine, Field Orientation, Artificial Neural Network.

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السيطرة المتقدمة على سرعة ماكنة حثية ثلاثية الأطوار بالاعتماد على توجيه الفيض الدوار والساكن وباستخدام تقنية الشبكة العصبية الاصطناعية

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المستخلص

يتضمن البحث مقترحاً لتصميم طريقة لتحسين عملية التحكم والتقييم للسرعة في منظومة سوق ماكنة حثية ثلاثية الطور اعتماداً على تقنية توجية الفيض من دون متحسس للسرعة، إذ تم نمذجة دائرة تقييم السرعة وبنائها بالاعتماد على تقنية تقييم فيض الجزء الساكن للماكنة مع دمج الشبكة العصبية الاصطناعية لتحسين عملية التقييم. أما تمثيل دائرة السيطرة فتم بالاعتماد على استراتيجية تقييم فيض الجزء الدوار للماكنة مع استخدام تقنية تضمين عرض النبضة لحزمة الهسترة للتيار كإشارات سوق للماكنة. إن طريقة السيطرة هذه تسمح بمرور تيار جيبي للماكنة يولد سرعة وعزم بأقل تموج ممكن مع تقليل التلوث السمعي للماكنة خصوصاً في حالة منطقة إضعاف الفيض عند السرع العالية والمنخفضة.

وقد بينت نتائج التمثيل الحاسوبي باستخدام برنامج التمثيل والمحاكاة الأداء المنضبط والمقبول للماكنة في حالة اتجاه الدوران الأمامي والعكسي وعمل الماكنة كمحرك أو مولد، فعند عمل الماكنة كمحرك كان مدى السرعة (0 - ± 1250 rpm) لعزوم ميكانيكية تصل إلى (0-±7 N-m) في الظروف المستقرة والديناميكية.

الكلمات المفتاحية: ماكنة السيطرة، تحديد الموقع، شبكة الخطوط الحيادية المصنعة.

1) Introduction:

The Induction Motors (IMs) are used heavily in industry field due to their combined advantage of operational reliability, performance, and ruggedness and relatively low cost if compared with DC Motor [1]. Since (IMs) are highly inherent coupling systems, a conventional scalar control is slightly simple to implement because both torque and flux are function of voltage or currents and frequency, that gives sluggish response, ending to instability because of a higher order (fifth order) system effect [2]. Also the Volt./Hz control performance is not satisfactory because the rate of change of voltage and frequency has to be low which can cause a transient change in the current results drastic problems during sudden acceleration or deceleration [3,4]. Therefore; in a vector control technique (direct and indirect) field oriented control (FOC). The IMs can be controlled as a separately excited DC motors and solve the IMs control problems for dynamic performance, for energy and efficiency to give it properties like DC machines. The FOC approach is objective to synthesis a robust linear controller and allows sinusoidal current drive the motor with generating low torque ripple with rotor speed and parameters variation taking into minimizes an acoustic noise for field-weakening operation for high-speed [2,3,5].

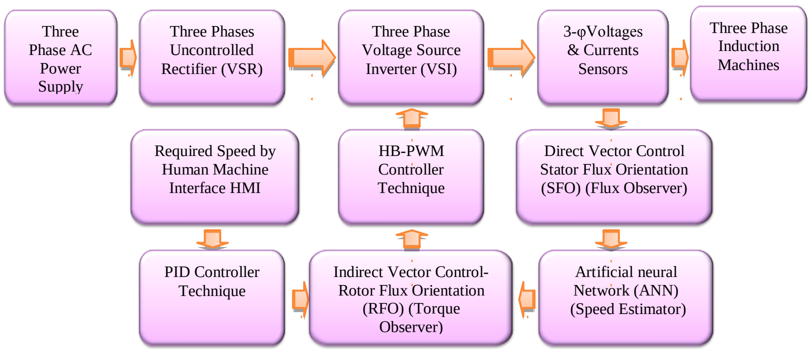

Fig (1): Generic Block Diagram Three Phase Induction Machine Drive of Proposed System

In this paper, a system configuration is presented for three phase IM having AC\AC converter which consist of uncontrolled rectifier - pulse width modulated IGBT inverter set with smoothing inductance and shunt capacitance as DC filter are proposed. An Artificial Neural Networks have been applied successfully to enhance the AC drive system including control, estimation of speed and torque characteristics. Simulation results for the proposed drive system are presented and proved to carry out insure high performance [6].

2. Field Orientation Control and ANN of Induction Motor:

A Field Oriented Control (FOC) is becoming an industry standard for controlling induction machines in high performance drive applications. It's used to accomplish decoupled control of flux and torque. It has two requirements [3]:

- An independent control of armature current to overcome the effects of armature, leakage inductance and induced voltage.
- An independent control of constant value of stator and rotor flux.

However, shaft mounted speed sensors in conventional Vector control (VC) drives degrade the system reliability and require special attention to electrical noise [3, 6]. Therefore; in this paper, the ANNs approach had been proposed to develop accurate rotor speed estimation by FOC technique and to eliminate problems related to shaft mechanical speed transducers, hence a design of control system is realized by the following implemented methodologies:

- Direct Stator Flux Orientation (DSFO).
- Artificial Neural Networks (ANNs).
- Indirect Rotor Flux Orientation (IRFO).
- Hysteresis Band Current Controller Technique (HBPWM).

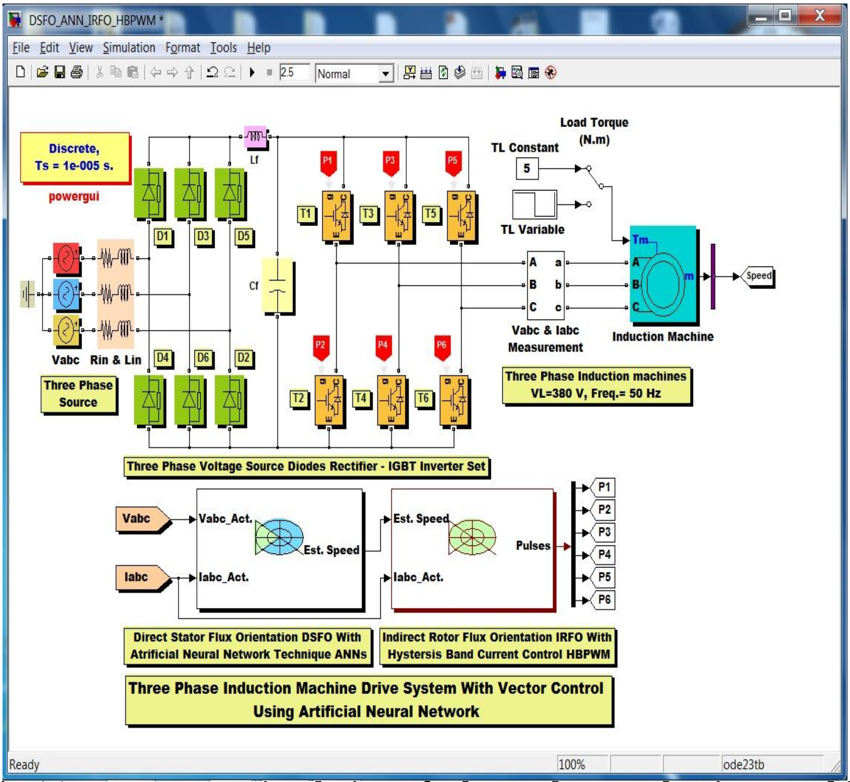

Fig (2): Computer Simulation of Three Phase Induction Machine Drive System Utilizing Speed Sensor less Field Orientated Control with ANN

2.1 Direct Stator Flux Orientation (DSFO) Using ANNs:

A DSFO control and senseless control rely heavily on accurate flux estimation; this method is most often used for senseless control, because the flux observer is used to estimate the synchronous, slip and rotor speed or angle estimation to removal mechanical speed sensors [3]. The stator flux estimation can be classified to Current Model Flux Observer (CMFO) and Voltage Model Flux Observer (VMFO) [2, 3]. In these methods, the flux vector estimation accuracy is affected by the stator resistance variation only that cause to non-robust control at field-weakening region near low speed. ANN is an alternative approach to overcome the problems related with leakage inductance, induce voltage moreover stator resistance variation under this cases, and to simplify the speed estimation and control with DSFO strategy [3, 8].

2.2 Artificial Neural Networks ANNs:

An Artificial Neural Network (ANN) for rotor speed estimation of induction machine is proposed to develop and enhance DSFO performance as shown in fig (3). Since ANNs have the merits of extremely fast parallel computation, immunity from input harmonic ripple, learning capability, generalization and error tolerance characteristics due to distributed network intelligence [6, 8]. The ANN system valuable and control involve the offline training of arbitrarily large (many hidden layers and nodes in each layer) feed forward sigmoid.

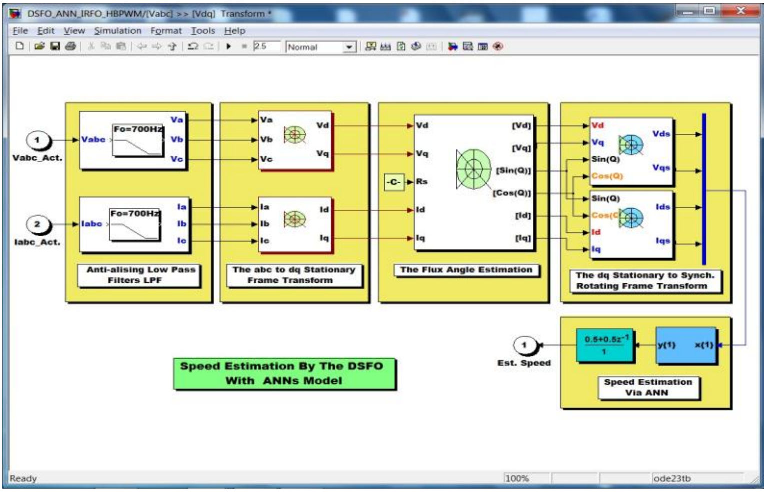

Fig (3): Simulation Model of Speed Sensor less Based on DSFO with ANN Technique

The ANNs are used for the back propagation neural (BPN) algorithm training algorithm. This offline training has done using large amount of data taken directly from the process to be identified and feed-forward network to learn a training set input-output pairs. The outputs of the nodes of each layer affect only the outputs of the next layer, in back propagation stage, the difference (errors) between real output values (target) and actual output values and sum of their square are calculated and minimized by Least Mean Square Error (LMS)[8,9].

2.3 Mathematical Model in D-Q Stationary Reference Frame:

In this model as shown in fig (3) by the DSFO method, a feedback signals required simulating the proposed scheme. Anti-aliasing low pass filters are used with stator phase voltages and currents feedback signals from three phase induction machine model to remove harmful high frequency waveforms. In this paper, Butterworth type, 2nd-order filters are used with cut off frequency. The stator winding quantities (phase voltages and currents) are converted to Direct-Quadrate stationary reference frame components by (3φ/2φ) transformation as in the following equations [6, 10, 11]:

$$Vq = \frac{2}{3}v_a - \frac{1}{3}v_b - \frac{1}{3}v_c \text{.......(1)} \qquad Vd = -\frac{1}{\sqrt{3}}v_b + \frac{1}{\sqrt{3}}v_c \text{..........(2)}$$

$$Iq = \frac{2}{3}i_a - \frac{1}{3}i_b - \frac{1}{3}i_c \text{............(3)} \qquad Id = -\frac{1}{\sqrt{3}}i_b + \frac{1}{\sqrt{3}}i_c \text{.............(4)}$$

By utilization of space phasor theory, a stator flux angle (θ_e) with respect to the d-axis of a stationary reference frame can be estimated from stator flux in stationary d-q frame, hence ψ_{ds}^{s}, ψ_{qs}^{s} and thus ψ_s can be obtained by integration of phase voltage and voltage drop on the stator resistance as in the following equations [6,10]:

$$\psi q = \int (Vq - Rs.Iq)dt ....(5) \qquad \psi d = \int (Vd - Rs.Id)dt ....(6)$$

$$\sin(\theta e) = \frac{\psi q}{|\psi s|} ....(7) \qquad \cos(\theta e) = \frac{\psi d}{|\psi s|} ....(8)$$

Where:

$$\psi s = \sqrt{\psi d^2 + \psi q^2} \quad ....(9)$$

The stator winding quantities of voltages and currents quantities can be converted from stationary (d^s-q^s) to synchronous rotating reference frame (d^e-q^e) as in the following equations [2, 10, 11]:

$$Vqs = Vq\cos(\theta e) - Vd\sin(\theta e)......(10)$$
$$Vds = Vq\sin(\theta e) + Vd\cos(\theta e).......(11)$$

$$Iqs = Iq\cos(\theta e) - Id\sin(\theta e)........(12)$$
$$Ids = Iq\sin(\theta e) + Id\cos(\theta e).........(13)$$

Since stator winding voltages and currents (Vds, Vqs, Ids, Iqs) magnitudes at synchronous rotating reference frame are inherently function of rotor speed, flux and load torque variation response. So these quantities are designed to be ANN inputs, thereby rotor speed is estimated by ANN output. A digital filter (Finite Impulse Response FIR-Direct form) is used to improve the pattern recognition perform of (BPN) which used logistic sigmoid as activation transfer function [13].

2.4 The Indirect Rotor Flux Orientation (IRFO):

A Rotor Flux Control is used in the Indirect Rotor Flux Orientation (IRFO) approach as shown in fig (4), that is obtained from the simple dynamic estimator for fast response, ease and parameter sensitive [2,12]. This method is used in high performance motion control drive applications which become an industry standard for controlling induction machines, the main objective of the IRFO control is to monitor the torque and the flux separately, as in direct current DC machines. These relations suggest that flux and torque can be controlled independently by d-q axis currents in rotating reference frame synchronously with the rotor flux space vector. The slip frequency is a satisfied coupling between stator current and rotor flux, therefore; to adjusting the flux while acting on the direct component and torque while acting on the quadrate component of the stator current. Thereby the IRFO control can be implemented for constant stator and air-gap flux orientation [2, 3, 6].

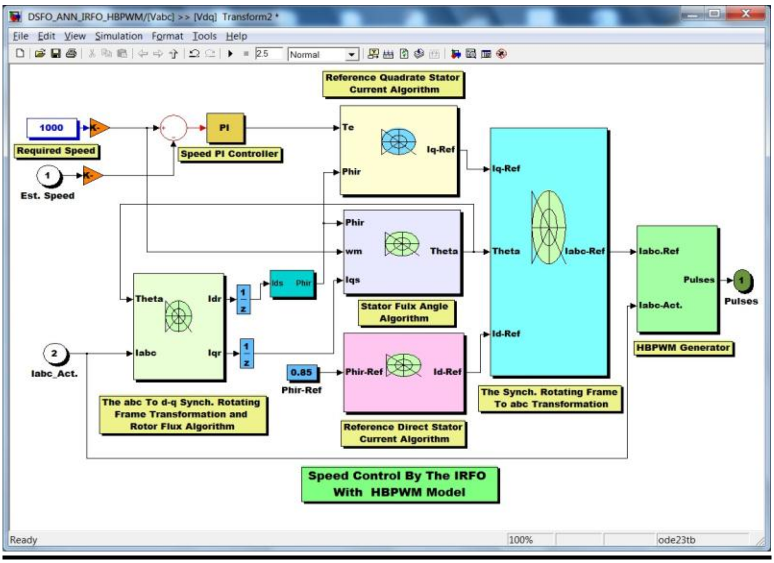

Fig (4): Simulation Model of Speed Sensor less Based on IRFO with HBPWM Technique

2.5 Rotor Flux Orientation Mathematical Simulation Model:

Indirect Vector Control (IVC) manner essentially is the same as the direct vector control, except the unit vector signals cos(θ) and sin(θ) which are generated in the feed forward method. The IVC fundamental principle is explained with phasor diagram as shown in fig (5) [2, 5, 6]. The (d^s-q^s) axes of stationary reference frame are fixed at stator while stationary reference frame (d^r-q^r) axes of synchronous rotation frame are moving at rotor speed (ω_r). The (d^e-q^e) are rotating ahead of (d^r-q^r) axes by positive slip angle (θ_{sl}) corresponding to slip frequency (ω_{sl}). Since the rotor pole is directed on (d^e) axis and ($\omega_e=\omega_r+\omega_{sl}$), hence the unit vector equation of the synchronous angle is [2, 10]:

$$\theta e = \int (\omega e) = \int (\omega r + \omega sl)dt = \theta r + \theta sl ......(14)$$

For decoupling control, derivation of IVC equation for total rotor flux (ψ_r) directed on (d^e) axis is as the following equations [2, 10,11]:

$$Lr / Rr \frac{d\psi r}{Lm} + \psi r = Lm.Idr ....(15)$$

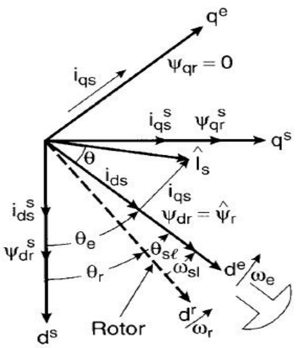

Fig (5): The Phasor Diagram by IRFO

When rotor flux is constant, it is usually in the form equation is [2,10]:

$$\psi r = Lm.Idr ....(16)$$

The rotor speed (ω_r) is estimated by ANN and the slip speed (ω_{sl}) can also be calculated as [2, 10, 11]:

$$\omega sl = Lm.Rr.Iqr / Lr.\psi r ....(17)$$

By substituting equations (16), (17) and the estimated speed via ANN in equation (14), gives the angle (θ_e). So the abc transformation of actual three phase currents are converted to synchronous rotating frame by a unit vector according to the following equations [2, 10]:

$$Idr = (2/3)[Ia.\sin(\theta e) + Ib.\sin(\theta e - (2\pi/3) + Ic.\sin(\theta e + (2\pi/3)] \text{.......(18)}$$

$$Iqr = (2/3)[Ia.\cos(\theta e) + Ib.\cos(\theta e - (2\pi/3) + Ic.\cos(\theta e + (2\pi/3)] \text{.....(19)}$$

2.6 Hysteresis Band Current Controller PWM Strategy:

The hysteresis band Pulse Width Modulation HBPWM is principally an instantaneous feedback current control method where the actual current continually tracks a command current within the hysteresis band. The control circuit generates a reference current wave of desired magnitude and frequency and it is compared with the actual phase current wave [2]. In this paper, the abc three phase reference currents are computed by IRFO method by reference currents of synchronous rotating frame according to following equations:

$$Idr\_ref = \frac{\psi r\_ref}{Lm} \text{...................(20)}$$

$$Iqr\_ref = \frac{4.Lr.Te}{3.p.Lm.\psi r} \text{..............(21)}$$

Where a developed torque (Te) is processed by PI controller, therefore; the abc three phase reference currents are [11, 12, 13]:

$$Ia\_ref = [Idr\_ref.\sin(\theta) + Iqr\_ref.\cos(\theta)] \text{.................(22)}$$

$$Ib\_ref = [Idr\_ref.\sin(\theta - 2.\pi/2) + Iqr\_ref.\cos(\theta - 2.\pi/3)] \text{.................(23)}$$

$$Ic\_ref = [Idr\_ref.\sin(\theta + 2.\pi/2) + Iqr\_ref.\cos(\theta + 2.\pi/3)] \text{................(24)}$$

When the actual current exceeds a prescribed hysteresis band, the upper switch in the half-bridge of the inverter is turned off and the lower is turned on. As the current crosses the lower band edge, the switches will be reversed .The output voltage transition from (+0.5 to- 0.5) Dc Voltage and current starts to decay [2]. The current pulses depend on the machine parameters and the hysteresis-band (HB) of hysteresis controller. the reconstructed currents are affected by HB by adjusting appropriate value; in this paper, The selection of appropriate HB for a given application had done by trial and error to obtain less current ripple, So a chosen HB is (0.25) of motor rated current. The reconstruction of line currents is possible over the entire operating speed [10].

3. Simulation and Results:

The three phase induction machine drive system having speed sensor less technique utilizing direct stator flux oriented method integrated with artificial neural network (DSFO-ANN), and speed controller using indirect rotor flux oriented method with hysteresis band current control pulse width modulation (IRFO-HBPWM) strategy are designed, performed, simulated and modeled by Mat lab\Simulink program. In this work, the ANN is trained offline to determine the suitable value of machine speed with accuracy and error bound equal to (1e-4) and the neurons number in hidden layer is specified as (50) neurons. The closed loop PI-Controller design is tuned to achieve speed control with minimum transient and over shoot. The PI controller parameter gains are proximity (Kp=5) (Ki=150). The simulation results in the fig (6-a) illustrates the speed response of induction machine with fixed torque (5 N-m) and comparison with desired rotor speed by human machine interface (HMI), estimated rotor speed by (DSFO-ANN) circuit and actual rotor speed by power system block sets PSB_S model from (500, 1250) rpm to zero speed operation at starting time, (1, 2 Sec) receptively. Fig (6-b) shows speed response with toque variation from (7, 5 N-m) at (0.75, 1.5 sec) receptively. Fig (7-

a&b) illustrates the induction machine rotor speed (estimated, actual) and torque response at forward and reverse motoring operation. At this mode, the fig (8-a) and fig (8-b) present the stator, (equation 9) rotor flux (equation 16) and the d-q stator voltages as ones of ANN inputs (equations 10,11) in synchronous rotating reference frame. The others of ANN inputs are d-q stator currents (equations 12,13) shown in fig (9-a) and d-q rotor currents (equations 18, 19) in same frame in fig (9-b).The three phase driven currents of machine during (±1000 rpm) speed reversal condition at (1 sec) and HB is (0.25) are shown in Fig (10-a) for actual currents and Fig (10-b) for reference currents of (IRFO-HBPWM) circuit (equations 22, 23, 24).

At forward and reverse generating operation (2^{nd} & 4^{th} quadrants) of induction machine, the fig (11-a&b) show estimated, actual rotor speed and torque response of its. The fig (12-a&b) and (13-a&b) present (the stator with rotor flux); the d-q stator voltages, the d-q stator currents and the d-q rotor currents responses at synchronous rotating reference frame respectively.

When induction machine during speed reversal from (±1000 rpm) at (1 sec) by prime mover (input mechanical torque), three phase inversion currents comparing with reference currents of control circuit as shown in fig (14-a) and fig (14-b) respectively.

4. Conclusions:

In this work, a speed control of a three phase induction machine drive system based on direct stator flux oriented DSFO integrated by ANN to improve speed estimation and indirect rotor flux oriented IRFO by hysteresis band current control HB-PWM for speed control is presented. The proposed model is constructed in this combination is designed to control and drive the motor in high and low speeds with any disturbance or varying in the mechanical load. Computer simulation results are designed and modeled with Mat lab\Simulink program showed that speed control approach will provide a wide range control for AC drive speed from zero speed up to ±1250 rpm, and regulated quickly versus load torque changing from (0 up to ±7 N-m). The control system combination provides an optimization speed control response, achieving smooth starting, acceleration, deceleration and robustness control at less change of desired speed if comparing with conventional AC drive controllers (i.e. reliability, accuracy and good performance).

5. List of Symbols and Units:

Va,b,c & Ia,b,c : Three phase stator voltages [V] and currents [A] respectively.

Vd,Vq & Id,Iq : Direct and Quadrate stator voltages [V] and currents [A] in stationary reference frame respectively.

Ids,Iqs & Idr,Iqr : Direct and Quadrate stator and rotor currents in synchronous rotating frame respectively [A].

Lm : Mutual inductance [H], Ls : Stator inductance [H], Lr : Rotor inductance [H].

Rs : Stator resistance [Ω]. Rr : Rotor resistance [Ω].

$\psi d,\psi q$: D-Q stator flux in stationary reference frame [wb] respectively.

ψs & ψr : Stator flux and rotor flux [wb] respectively.

$\omega e,\omega sl$ & ωr :Synchronous speed, slip speed and rotor speed respectively [rad\sec].

Te : Developed Torque [N-m].

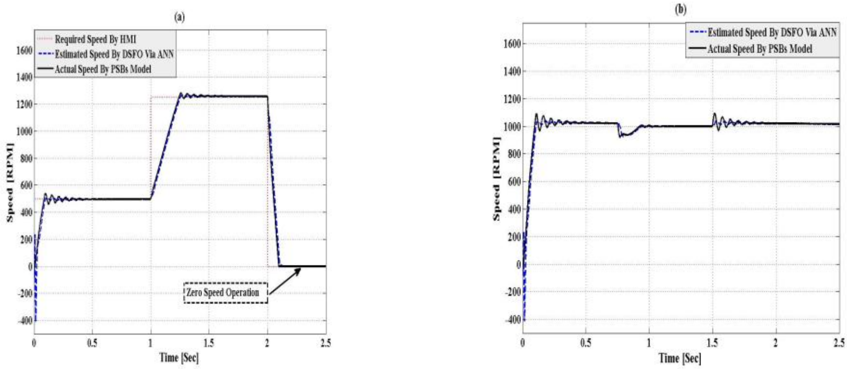

Fig (6): Computer Simulation results of speed response of Induction Machine Drive System:
(a) Speed Variation with fixed load torque
(b) Torque Variation with Constant Speed torque

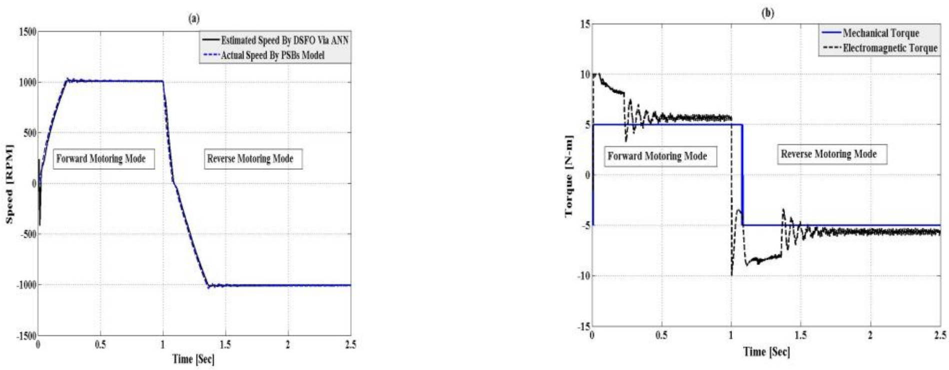

Fig (7): Computer Simulation Results of the Induction Machine Drive System at Motoring Operation:
(a) Speed Response
(b) Torque Response

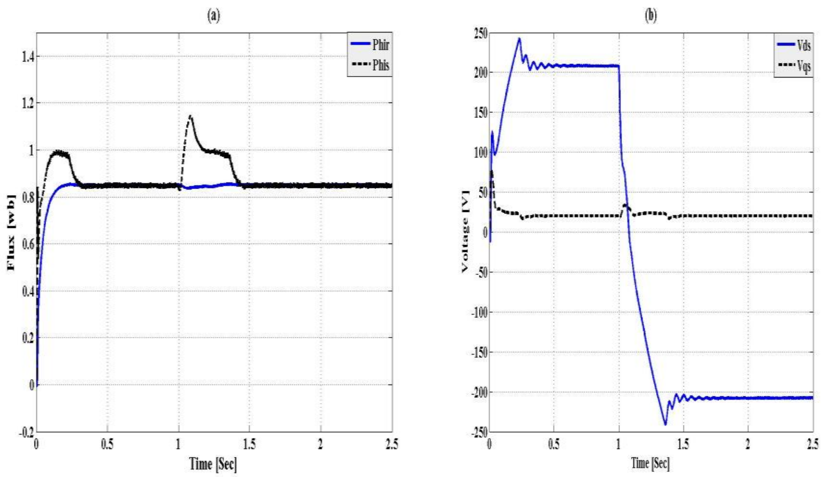

Fig (8): Computer Simulation Results in synchronous rotating reference frame at Motoring Operation:
(a) Stator and Rotor Flux.
(b) Stator d-q Voltages.

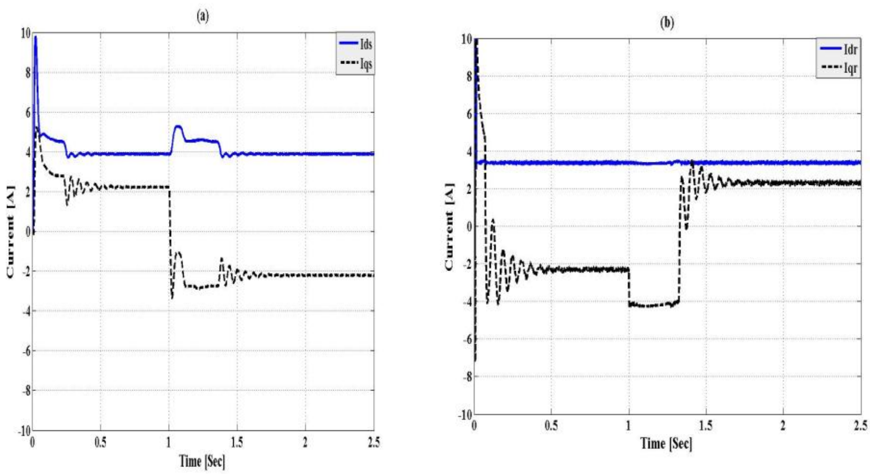

Fig (9): Computer Simulation Results in synchronous rotating reference frame at Motoring Operation:
(a) Stator d-q Currents.
(b) Rotor d-q Currents.

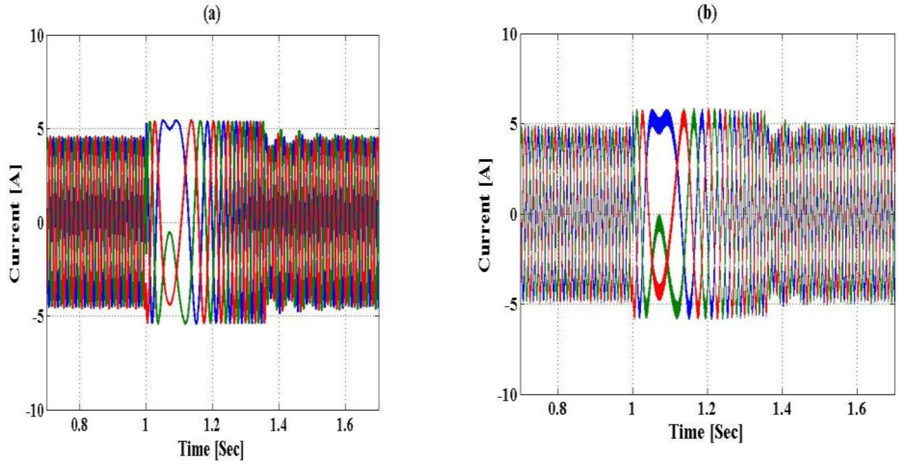

Fig (10): Computer Simulation Results of Three Phase Machine Currents at Motoring Operation:
(a) Actual (driven) Current of Machine.
(b) Ref. Currents of Control Circuit.

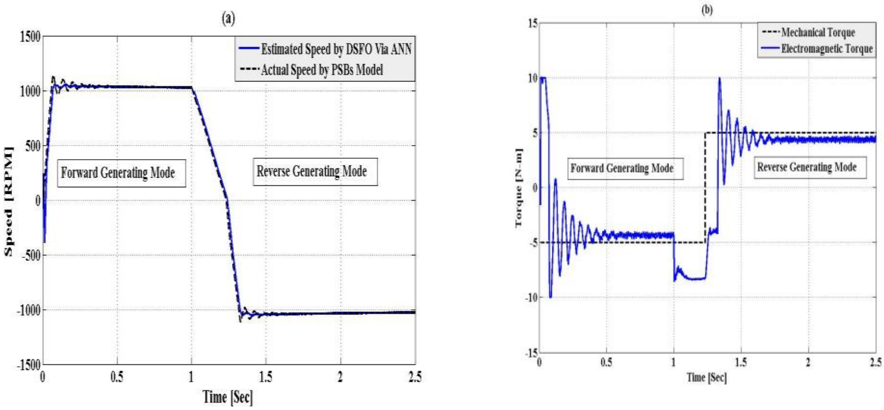

Fig (11): Computer Simulation Results of the Induction Machine Drive System at Generating Operation:
(a) Speed Response.
(b) Torque Response.

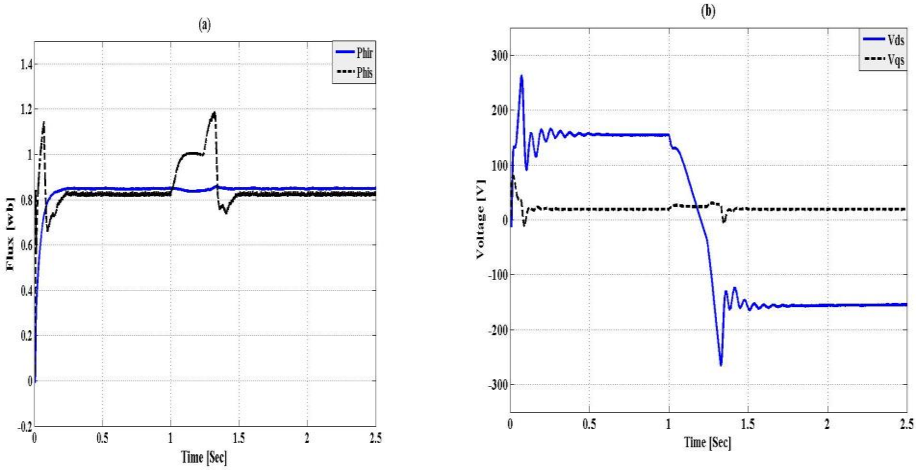

Fig (12): Computer Simulation Results in synchronous rotating reference frame at Generating Operation:
(a) Stator and Rotor Flux.
(b) Stator d-q Voltages.

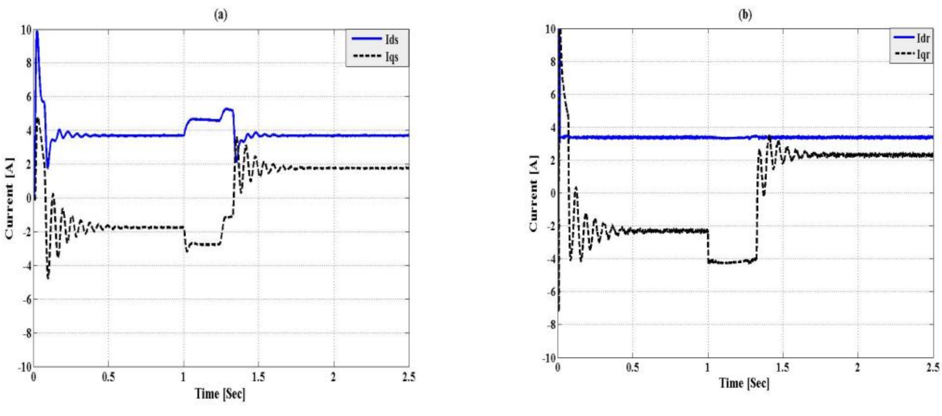

Fig (13): Computer Simulation Results in synchronous rotating reference frame at Generating Operation:
(a) Stator d-q Currents.
(b) Rotor d-q Currents.

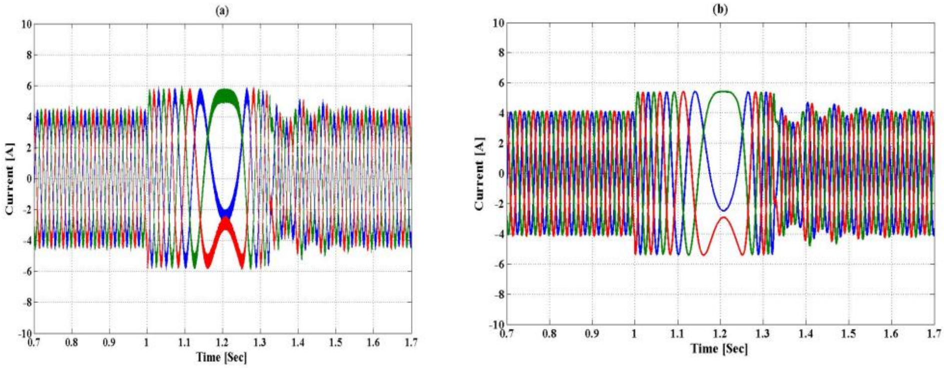

Fig (14): Computer Simulation Results of Three Phase Machine Currents at Generating Operation:
(a) Actual (inversion) Current of Machine.
(b) Ref. Currents of Control Circuit.

6. Appendix (Drive System Specifications):

I. Induction Machine:

Three phase, squirrel-cage type, BHP = 1.5 [hp], phase voltage = 220 [V], freq. =50 [Hz], Pole pairs =2, Efficiency = 67 %, Power factor = 0.7, Moment of inertia= 0.0075 [$Kg.m^2$],

Rin = 0.05 [Ω] Lin = 0.005 [H], Rs = 5.1975 [Ω] Rr = 6.7333 [Ω], Ls = 0.2679 [H], Lr = 0.2679 [H], Lm = 0.2512 [H]. n_{noload}= 1486 [rpm].

II. Analogue Filters:

Low pass filters, 2nd-Order of Butterworth type, High cut-off frequency = (700 & 300) [Hz].

III. Digital Filter:

Finite Impulse Response FIR Type, coefficient structure: Direct form, numerator coefficients [0.5 0.5], sampling time (Ts) =1e-5 sec.

IV. DC-Filter Unit:

V_{DC} = 513 [V], Shunt capacitance (C) = 1000μ [F], Smoothing inductance (L) = 500μ [H].

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